T: # LAND MANAGEMENT PLANNING

GOAL PROGRAMMING FOR LAND MANAGEMENT PLANNING BASICS AND PROCEDURES

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GOAL PROGRAMMING FOR LAND MANAGEMENT PLANNING

Contact: Systems Application Unit
for Land Management Planning
U. S. Forest Service
3825 East Mulberry
Fort Collins, Colorado 80521

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ABSTRACT

Decision-making systems for natural ecosystems must be capable of handling the immense data base and the complex decision environment inherent in management of public lands. Competition between resource users for the fixed natural resource base causes our public lands to be a very productive breeding ground for conflict. This conflict arises from physical or normative constraints or desires. A system for resource allocation must consider multiple conflicting goals arising from the physical capabilities of the ecosystem and the normative desires of the public. Goal programming, a computer oriented extension of linear programming, provides an efficient mechanism for evaluating conflicting goals while considering the ecological capabilities of natural ecosystems.

The inherent characteristics of both linear and goal programming are presented. The advantages of goal programming over linear programming are demonstrated through simple examples.

INTRODUCTION

Decision making is a basic process common only to higher forms of animal life. Man is a decision maker. He attempts to obtain the maximum possible attainment of goals within a given set of constraints. He attempts to select the courses of action from a set of alternatives which will achieve his objectives.

The primary incentive for natural resource decision making is the basic human and economic problem of satisfying unlimited human desires with limited resources. Decision makers are hindered by an environment of conflicting interests, incomplete information, insufficient resources, and limited ability to analyze the complex environment.

Attempting to aid decision making, much emphasis has recently been placed on decision analysis. Decision analysis is a quantitative, computer approach which has three well defined characteristics which are essential for resource management:

(i) identification of organizational goals and environmental constraints,

(ii) explicit analysis of relationships between ecosystem components and alternatives for goal attainment, and

(iii) guides to the evaluation of optimal courses of action.

The final evaluation of a decision analysis technique should be measured on the degree to which goals or objectives of the system are achieved.

The purpose of this report is to compare two decision analysis techniques: linear programming and goal programming. Discussion and comparison of both methods and detailed examples are presented.

LINEAR PROGRAMMING

Economic theory and decision analysis are based on the assumption of optimization, whether it be in terms of profit maximization, cost minimization or the attainment of another goal. The evaluation of management alternatives, to arrive at an optimal combination of alternatives and products, is commonly based on a single objective, optimization of monetary values. For this single goal system, linear programming is sufficient. Linear programming techniques have been applied to natural resource decision processes in the past few years and application is becoming more frequent (Wardle, 1965; Bare, 1971).

Linear programming has three basic characteristics:

(i) The decision variables constituting the decision environment are homogeneous and linear.

(ii) Constraints of limited resources or requirements are linear.

(iii) The objective function is homogeneous, linear and usually involves either maximization of profits or minimization of cost.

The standard formulation of linear programming is:

$$\text{Maximize or minimize: } Z = \underset{\sim}{c}\underset{\sim}{x}$$
$$\text{Subject to: } \underset{\sim}{A}\underset{\sim}{x} \leq \underset{\sim}{b} \text{ and } \underset{\sim}{x} \geq 0 \qquad (1)$$

where Z is the value of the objective function, c is a row vector of the degree of attainment of the objective by alternatives, x is a vector of

alternatives (decision variables), b a vector of constraint values, and A a matrix of coefficients relating the alternatives to the constraints. The solution of a linear program provides a guide to the optimization of the single objective, Z.

A single goal objective is not the case in natural resource management, where varied opinions exist about the goals or the use of a specified land area and its associated natural resources. Are the goals to maximize profit, maximize resource usage, minimize environmental impact, or are they some combination of these plus many more? Recent environmental awareness has shown the need for re-evaluation of organizational objectives and the development of new goals.

Multiple conflicting objectives become evident. A meaningful form of natural resource decision analysis must be capable of handling multiple conflicting goals. Linear programming provided the basis for an improved programming procedure called goal programming, which is capable of handling these multiple conflicting goals.

GOAL PROGRAMMING

Goal programming can provide a simultaneous solution to a system of conflicting multiple objectives. The technique is, therefore, capable of handling decision problems that deal with a single goal with multiple sub-goals, as well as those having multiple goals with multiple subgoals. Charnes and Cooper first presented goal programming (1961).

The concept of goal programming evolved as a result of unsolvable linear programming problems and the desire to improve on the weak points of the conventional linear programming model formulation. The basic assumption of goal programming is "whether goals are attainable or not, an objective may be stated in which optimization gives a result which comes 'as close as possible' to the indicated goals", (Lee, 1972).

The standard goal programming formulation is presented in Equation (2) and follows the development by Lee (1972).

$$
\begin{array}{lll}
\text{Minimize:} & \underset{\sim}{Z} = \underset{\sim}{d}^{-} + \underset{\sim}{d}^{+} & \\
\text{Subject to:} & B\underset{\sim}{x} + \underset{\sim}{d}^{-} - \underset{\sim}{d}^{+} = \underset{\sim}{h} & \\
 & A\underset{\sim}{x} \leq \underset{\sim}{b} & \qquad (2) \\
\text{and} & \underset{\sim}{x},\ \underset{\sim}{d}^{-},\ \underset{\sim}{d}^{+} \geq 0 &
\end{array}
$$

where $\underset{\sim}{d}^{-}$ and $\underset{\sim}{d}^{+}$ are vectors of deviations from a vector of goal levels $(\underset{\sim}{h})$. The objective function $(\underset{\sim}{Z})$ is to minimize the deviational values of $\underset{\sim}{d}^{+}$ and $\underset{\sim}{d}^{-}$ to as near the desired goal level as possible. When d_i^{+} and d_i^{-} are minimized the optimal attainment of goal "h_i" will be realized for a certain value of x (the vector of alternatives and products). The deviational variables d_i^{+} and d_i^{-} are complementary to each other. If d_i^{+} takes a non-zero value, d_i^{-} will be zero, and vice versa. Since at least one of the variables will be zero, $d_i^{-} \cdot d_i^{+} = 0$.

The objective function is composed of either a pair or a single deviational variable for each goal constraint. The decision maker must analyze each goal

(h_i) in terms of whether over or underachievement of the goal is acceptable. He can then assign deviational variables to the goals. If overachievement is acceptable, the positive deviation (d_i^+) can be eliminated from the objective function. On the other hand, if underachievement is satisfactory, the negative deviation (d_i^-) should not be included. Exact achievement of a goal requires both negative and positive deviations be represented in the objective function to achieve the ordinal solution.

The coefficient matrix, B, describes the degree of attainment each activity in $\tilde{x}$ contributes to each goal in $\tilde{h}$. The other components of Equation (2) are the same as in the linear program presented in Equation (1).

In most systems, goals are in competition for scarce resources. Once the present incompatible multiple goals is realized, a judgment about the importance of each individual goal must be made in such a way as to insure the most important goal will be achieved to the extent desired before the next goal is considered. To achieve the goals according to their importance, goal programming provides a means by which the negative and/or positive deviations about the goal may be ranked according to an ordinal priority ranking scale in order of preference for attainment of each goal level, h_i. The heart of the goal programming algorithm is the objective function which consists of at least two of three factors:

(i) deviational variables,

(ii) ordinal priority factors, and

(iii) weighed factors.

The deviational variables and the ordinal priority factors are always present in each objective function. The weights need not be assigned but are useful when needed.

Once deviational variables are determined, the next task is to assign the ordinal priority factors. Assume there are K rank of goals, the ordinal priority factors P_j(j j = 1,2,...,K), should be assigned to each of the deviational variables. Factors have the property $P_j >>> P_j + 1$, so that priority $P_j + 1$ is always less than P_j. For each P_j value the goal programming formulation will have a separate objective function, each of which must be commensurable within itself. To this point, the multiple objective functions of a goal programming problem consists of the deviational variables with their ordinal priority factors (P_j's). Every formulation must go at least this far.

Extra flexibility is added to the algorithm by the capability of adding weights to each priority--deviational variable combination ($P_j\ d_j^+$ or $P_j\ d_j^-$) within the same priority level. By definition the variables within a particular priority level must be commensurable, although between priority levels they need not be and, in fact, are usually not. Assume the weighting factors $\underset{\sim}{\partial}$'s can be assigned for weighting the deviation variables at the same priority level, ∂_i becomes weights for each priority level as shown in Equation (4) to form the row vector c.

$$\underset{\sim}{c} = (\partial_1 P_{j1}, \partial_2 P_{j2} \ldots, \partial_{2m} P_{j2m}) \quad \text{weighted priority factors} \qquad (3)$$

c becomes a vector of weighted priority factors. In other words, the P_{ji}(i=1,2,...,2m;j=1,2,...,k) are preemptive priority factors with the highest preemptive factors being P_1 and weight carrying ∂_i's (i=1,2,...2m) are real numbers. Let d be a 2m-component column vector whose elements are d_i^-'s and d_i^+'s such that:

$$d = (d_1^-, d_2^-, \ldots d_m^-; d_1^+, d_2^+, \ldots, d_m^+) \quad \text{deviational variables} \qquad (4)$$

In the multiple goal formulation, d becomes a vector of deviational variables for each of the multiple goals. Based on the above explanation, a goal programming problem involving multiple conflicting goals with weighted priority factors (∂P_{ji}) and deviational variables (d_i), can be formulated as:

$$\begin{aligned} &\text{Minimize} && \underset{\sim}{c}\underset{\sim}{d} \\ &\text{Subject to} && \underset{\sim}{B}\underset{\sim}{x} + \underset{\sim}{R}\underset{\sim}{d} = \underset{\sim}{b} \\ & && \underset{\sim}{x}, \underset{\sim}{d} \geq 0 \end{aligned} \tag{5}$$

where B and R are m x n and m x 2m matrices , respectively. In Equation (5), m is defined as the number of goals and n is the number of subgoal variables.

A summary to this point will reinforce the basic concepts of goal programming after which a few applications of goal programming will be outlined.

(i) Goal programming is an extension of the conventional linear programming model in which the optimum attainment of goals is achieved within a given decision environment. This environment is defined by the decision variables, constraints, objective function, priority factors, deviational variables and weights.

(ii) Decision variables are the real variables in the model. These values are arbitrarily assigned and changed in the search for the optimum set of values. The decision variables are related among themselves and to other variables by values which are specified by the environment. In the above models the decision variables constitute a vector of all management (production) and product (user) alternatives.

(iii) Constraints represent a set of linear relationships among resources and regulate the values of decision variables.

(iv) An objective function is a mathematical expression involving some variables in the model. The values are computed when the coefficients of all other variables are determined. The values in the objective function of goal programming will differ from those used in linear programming. Instead of maximizing profit or minimizing cost, we minimize the deviations between the desired goal levels and the actual level attained within the established constraint system.

SOLUTIONS

Both linear and goal programming problems can be solved by two methods:

(i) graphically, or

(ii) by use of the simplex algorithm.

For illustration purposes a linear programming problem will be formulated, solved geographically and then solved via the standard simplex technique. This same example will then be converted to a typical goal programming formulation and solved both graphically and via the modified simplex technique.

EXAMPLE 1

Linear Programming

A rancher has 80 acres of browse type mountain land located on hillsides throughout his ranch. He desires increased year-round forage for his cattle and wishes to know, based on estimated total revenue alone, how many acres are to be treated with each of two management treatments:

(i) chaining, fertilization and reseeding

(ii) spraying, fertilization and reseeding.

In addition to the 80 acres, the rancher will allow a maximum of 10 additional acres of stream bottomland to be treated. The rancher stipulates that no more than 70 acres can be chained and 45 acres sprayed. Treatment of chaining and spraying cannot exceed 90 acres. Assume the estimated yearly revenue resulting from chaining, clearing and seeding one acre is \$2.50 and the revenue from spraying, clearing and seeding one acre is \$1.50.

The linear programming model can be formulated as below:

$$\begin{array}{lrcrl} \text{Maximize} & Z = 2.50X_1 & + & 1.50X_2 & \\ \text{Subject to} & X_1 & & & \leq 70 \\ & & & X_2 & \leq 45 \\ & X_1 & + & X_2 & \leq 90 \\ & X_1, X_2 & & & \geq 0 \end{array} \qquad (6)$$

where

X_1 = the number of acres chained

X_2 = the number of acres sprayed.

Graphical Solution

The graphical solution to this problem is shown in Fig. 1.

Simplex Solution

Equation (6) can be expanded by adding slack variables and expressed as in Equation (7).

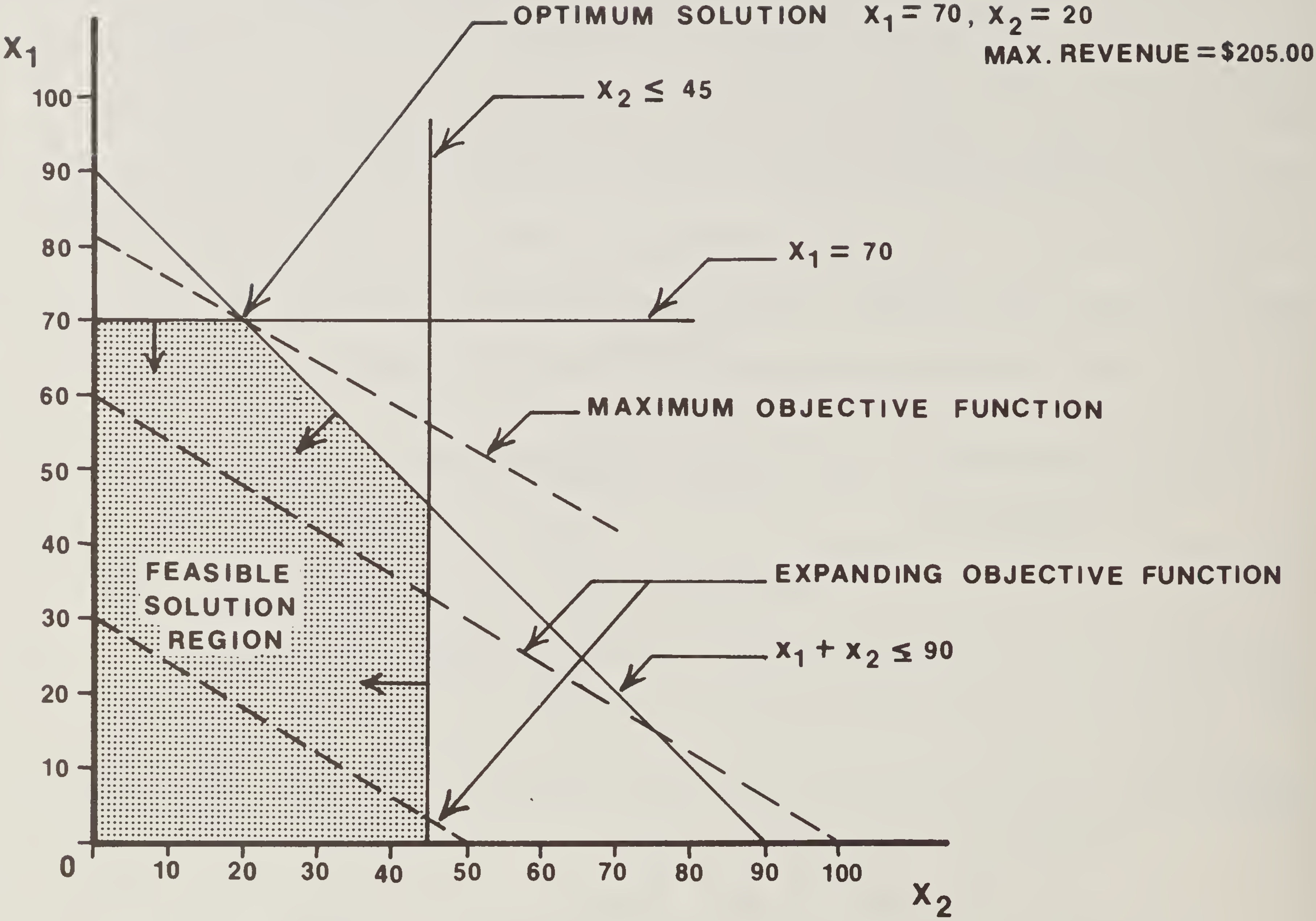

Fig. 1. Graphical Solution for Standard Linear Programming Model.

$$\begin{aligned}
\text{Maximize} \quad & Z - 2.50X_1 - 1.50X_2 + S_1 + S_2 + S_3 = 0 \\
\text{Subject to} \quad & X_1 + S_1 = 70 \\
& X_2 + S_2 = 45 \\
& X_1 + X_2 + S_3 = 90
\end{aligned} \tag{7}$$

Converting Equation (7) to matrix notation, the simplex algorithm can begin. Table 1 illustrates the initial feasible solution which is centered at the origin, point B, Fig. 1.

Table 1. Initial simplex tableau.

Row	X_0	X_1	X_2	S_1	S_2	S_3	C.S.	Basis
0	1	-2.50	-1.50	0	0	0	0	X_0
1	0	1	0	1	0	0	70	S_1
2	0	0	1	0	1	0	45	S_2
3	0	1	1	0	0	1	90	S_3

Enter X_1 because it will increase the objective function the greatest amount.

Row 1 70/1 = 70 therefore enter 70 units of X_1, exit S_1

2 45/0 = undefined

3 90/1 = 90

Row 1 becomes the tool row, because it is most constraining on X_1.

Once the tool row is identified, it is used to move the solutions along the boundary of the solution space shown in Fig. 1, until an optimum is reached at one of the extremes.

According to the simplex algorithm the pivot element in the tool row must be reduced to a 1 and the remaining elements in that column to zero. One such iteration is shown below.

	0	1	0	1	0	0	70	Row 1*
Row 0	1	-2.50	-1.50	0	0	0	0	
(Row 1) (2.50)	0	2.50	00	2.50	0	0	175.00	
	1	0	-1.50	2.50	0	0	175.00	Row 0*
Row 2	0	0	1	0	1	0	45	Row 2*
Row 3	0	1	1	0	0	1	90	
(Row 1) (-1)	0	-1	0	-1	0	0	-70	
	0	0	1	-1	0	1	20	Row 3*

These manipulations generate the results shown in Table 2. The column vector of zeros and the one in cell 2,2 of the matrix shown in Table 2 shows that X_1 is in the present solution.

Further iterations will not be discussed other than the tableau for each iteration. For further review of the simplex algorithm see Wagner, 1971.

Table 2. Tableau for first iteration.

Row	X_0	X_1	X_2	S_1	S_2	S_3	C.S.	Basis
0	1	0	-1.50	2.50	0	0	175.00	X_0
1	0	1	0	1	0	0	70	X_1
2	0	0	1	0	1	0	45	S_2
3	0	0	1	-1	0	1	20	S_3

Table 3. Final iteration tableau.

Row	X_0	X_1	X_2	S_1z	S_2z	S_3	C.S.	Basis
0	1	0	0	1.00	0	1.50	205.00	X_0
1	0	1	0	1	0	0	70	X_1
2	0	0	0	1	1	-1	25	S_2
3	0	0	1	-1	0	1	20	X_2

The optimal solution as shown in Table 3 is:

X_1 = 70 acres of land to be chained

X_2 = 20 acres of land to be sprayed

with an estimated revenue of $205.00.

Goal Programming

The above L.P. can be converted to the goal programming formulation shown in Equation (8).

$$
\begin{aligned}
&\text{Minimize } Z = P_1d_1^- + P_2d_4^+ + 5P_3d_2^- + 3P_3d_3^- + P_4d_1^+ \\
&\text{Subject to} \quad X_1 + X_2 + d_1^- \qquad -d_1^+ \qquad = 80 \text{ A)} \\
&\qquad X_1 \qquad + d_2^- \qquad = 70 \text{ B)} \\
&\qquad X_2 \qquad + d_3^- \qquad = 45 \text{ C)} \\
&\qquad X_1 + X_2 \qquad + d_4^- - d_4^+ \qquad = 90 \text{ D)} \\
&\qquad X_1, X_2, d_1^-, d_2^-, d_3^-, d_4^-, d_1^+, d_4^+ \qquad \geq 0
\end{aligned}
\tag{8}
$$

The decision variables X_1 and X_2, are expressed in terms of acres in the above model. The graphical illustration of the above model's constraints are shown in Fig. 2.

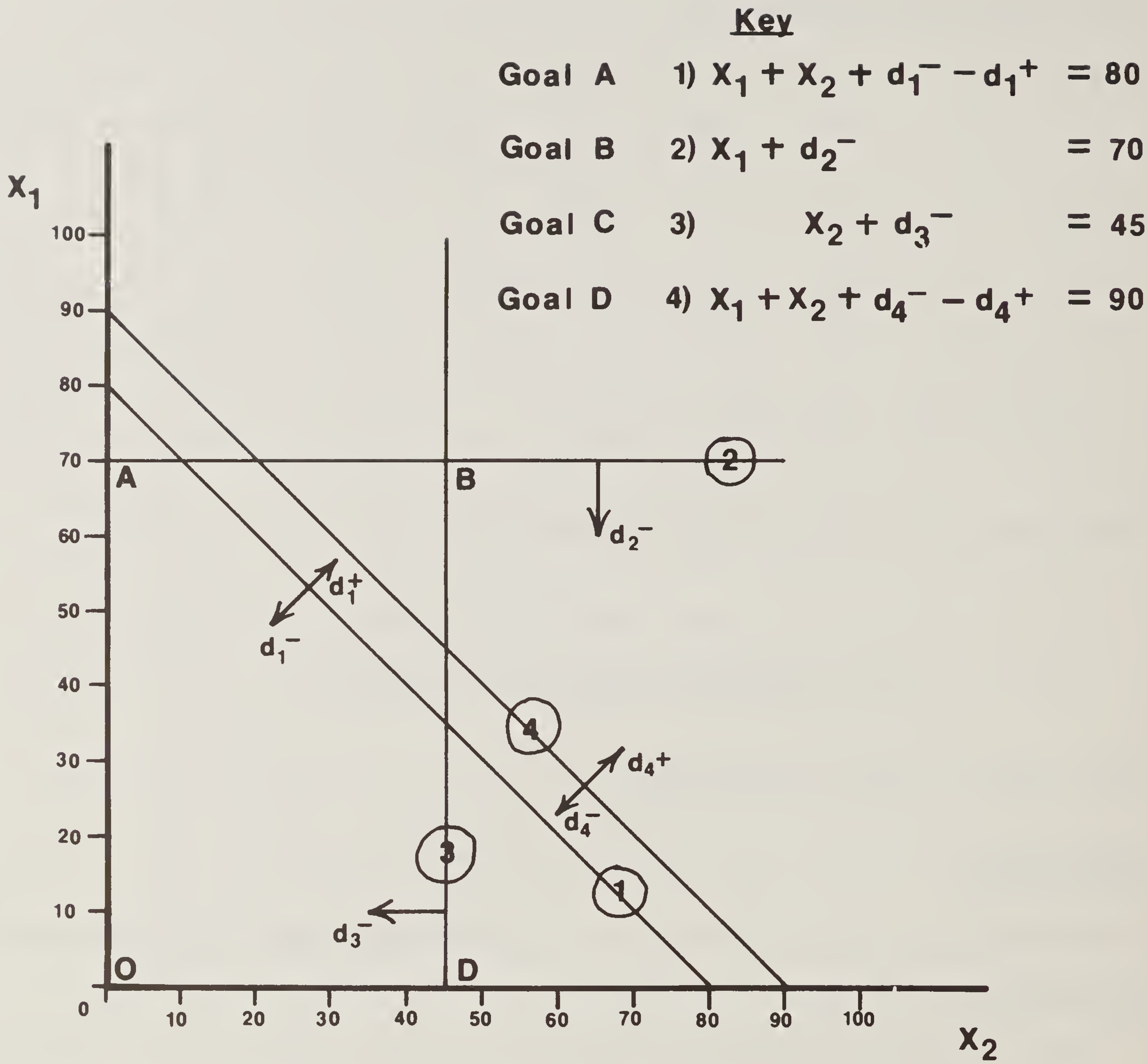

Fig. 2. Graphical Illustration of G. P. Constraints.

The objective of this model would be to minimize the deviation from the four goals, A, B, C, D. The goals are as follows:

Goal A) The treatments of chaining and spraying should include as close to 80 acres of browse land as possible.

Goal B) The treatment, chaining, should not exceed 70 acres of land. We wish to minimize the underachievement of this goal.

Goal C) The treatment spraying should not exceed 45 acres of browse land. We wish to minimize the overachievement of this goal.

Goal D) Chaining and spraying must be as close a total of 90 acres as possible: 80 acres browse, 10 acres bottomland. The rancher is willing to utilize more acreage if necessary.

The rancher assigns the following priorities to the goals as listed below:

P_1 - assigned to the minimization of the underutilization of the browse acreage (d_1^-)

P_2 - assigned to the minimization of the overutilization of the 10 acres of bottomland: 80 acres of browse land + 10 acres of bottomland (d_4^+)

$5P_3$ - assigned to the minimization of the underutilization of the 70 acres maximum available for chaining (d_2^-)

$3P_3$ - assigned to the minimization of the overutilization of the 45 acres maximum available for spraying (d_3^-)

P_4 - assigned to the minimization of the overutilization of the total browse acreage (d_1^+), because the priority one goal of minimizing the underutilization of the browse acreage may well force overutilization of the browse acreage.

Graphical Solution

Equation (7) shows that the use of acreage for both chaining and spraying are considered only on the basis of expected revenue. Table 4 outlines the rancher's estimated profit contribution of each acre. Noting the priority P_3 level the ranch wishes to weight the underachievement of Goal B with a five and the overachievement of Goal C with a three.

Table 4. Cost comparison.

Material	Profit/Acre	Ratio
Chaining	\$2.50	5
Spraying	\$1.50	3

The "optimal" solution lies in the area outlined by the rectangle ABDO as shown in Fig. 2. Now analyze the objective function. The first goal is to avoid underutilization of the browse land; therefore, minimize d_1^- to zero. This analysis leads to a reduction in the size of the solution space. The shaded area in Fig. 3 represents the feasible solution space resulting from the attainment of the priority one goal, minimization of underutilization of browse acreage capacity: $d_1^- = 0$.

The rancher's second goal is to limit the utilization of bottomland to 10 acres. Deviational variable d_4^+ is to be minimized to zero: $d_4^+ = 0$. The two most important goals will be achieved to the fullest as long as the situation occurs within the shaded area as shown in Fig. 4.

The third goal is to achieve maximum acreage utilization within the solution space defined by the higher priority goals. Because of the 5 to 3

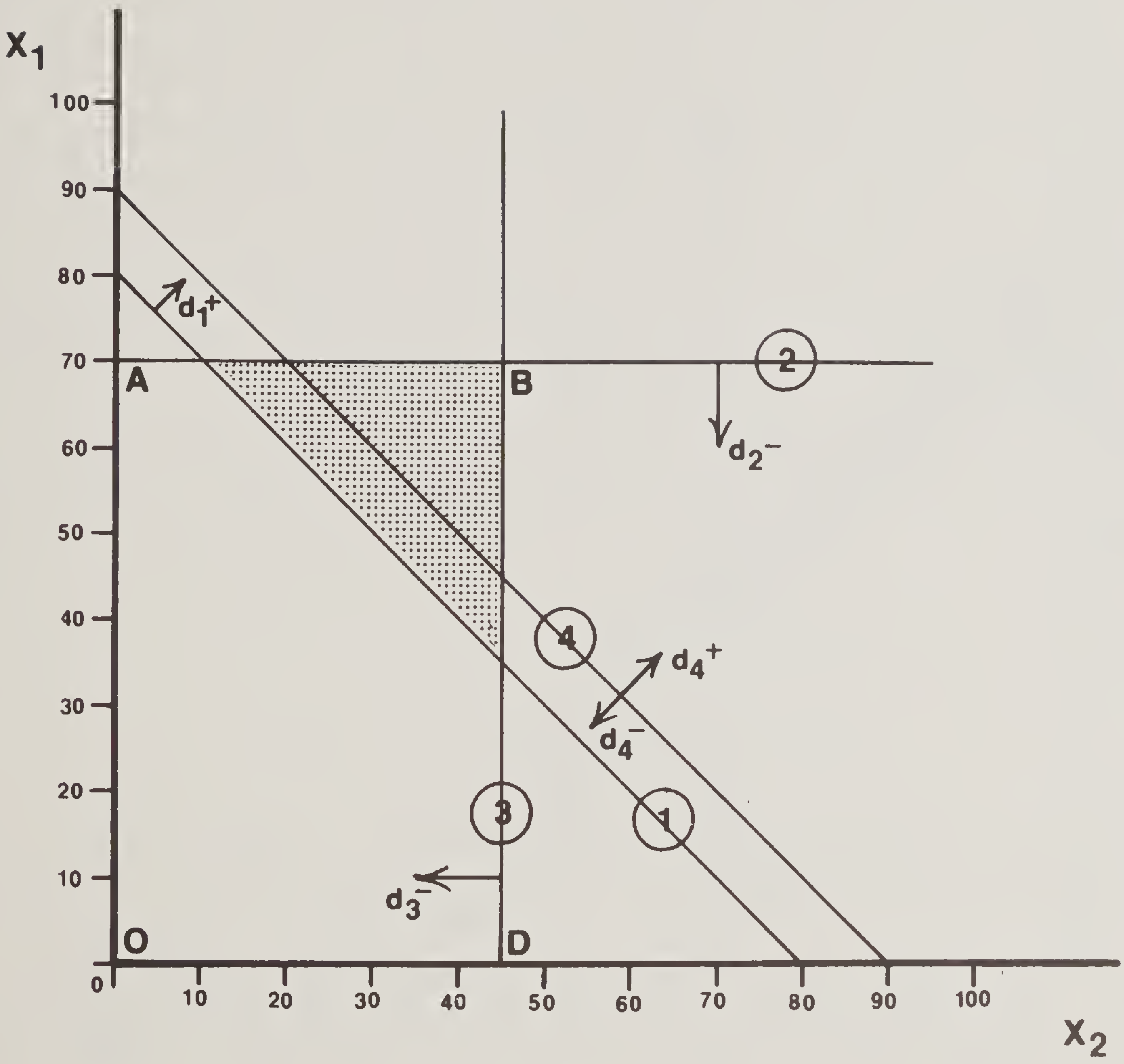

Fig. 3. Feasible Solution Space After Enforcement of Priority One Goal.

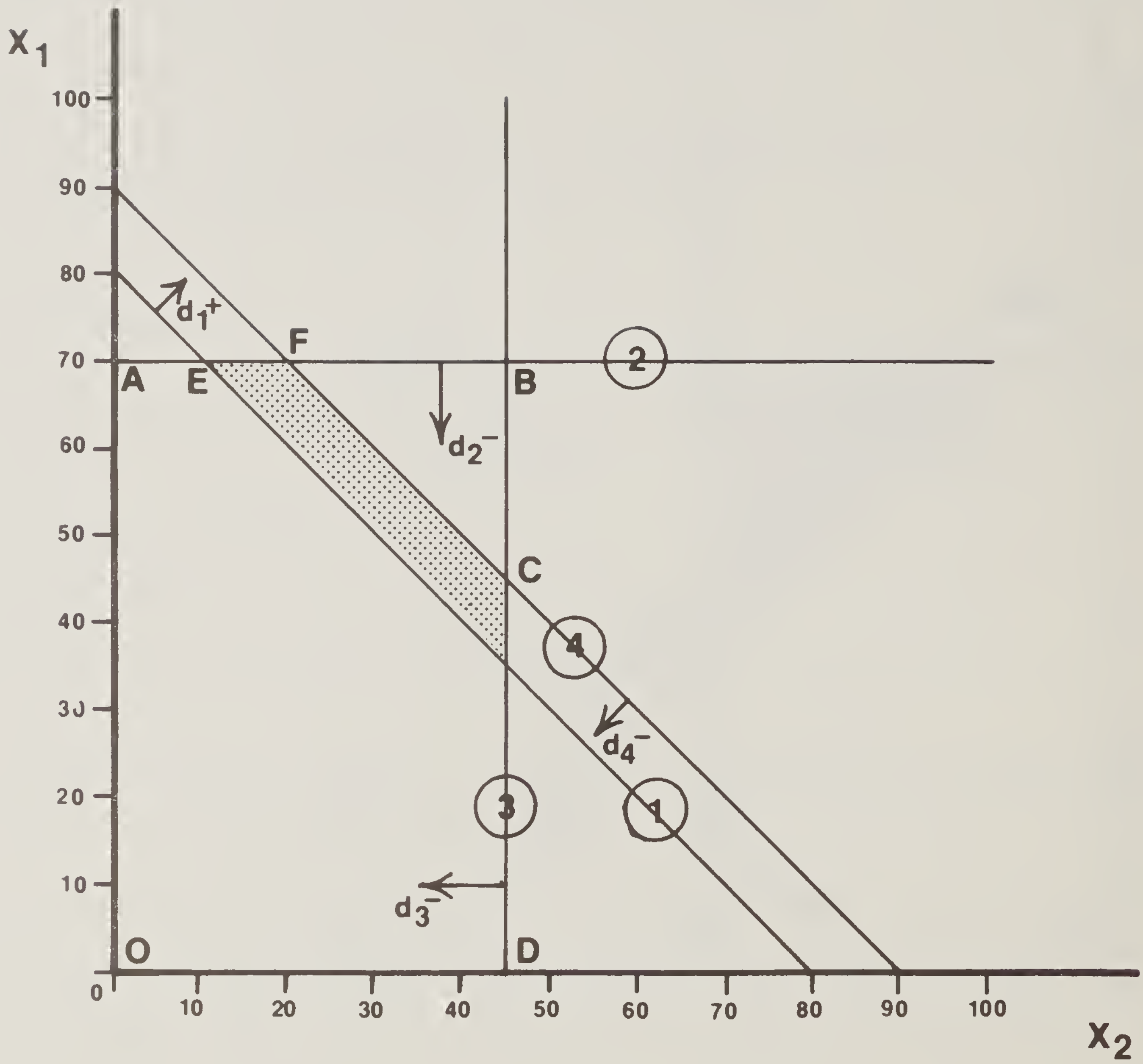

Fig. 4. Solution Space as Reduced by all Attainable Goal Priorities.

ratio of chaining (X_1) to spraying (X_2) the rancher should try to chain as many acres as possible: 70 acres as shown at any point on line segment EF in Fig. 4. It is impossible to achieve the maximum of 45 acres of spraying without reducing the 70 acre achievement of the chaining goal. If possible, the solution point would be at point B which lies out of the feasible solution region shown in Fig. 4. Point C would allow 45 acres to be sprayed and 45 acres to be chained. Both the goals of X_1 and X_2 cannot be obtained simultaneously. Realizing the maximum desired level of the priority 3 goal cannot be fully achieved, our object becomes the minimization of the non-attainment of Goal 3. This is done at point F as shown in Fig. 4. At this point the solution would be:

X_1 = chaining 70 acres

X_2 = spraying 20 acres

Point F, Fig. 4., yields the optimum answer under the pre-established constraint and priority assignments. If X_2 is increased, it will be at the expense of X_1. Goal B was given the P_2 priority ranking and therefore, has rank over Goal C. The further attainment of Goal C cannot be at the expense of Goal B thus forcing the point F to be optimum.

The fourth goal of the rancher was to minimize total bottomland acreage utilized. The bottomland acreage available for use is limited to 10 acres at solution point F. We could eliminate the use of the 10 acres by moving to the point E on Fig. 4, but would sacrifice 10 acres of the land that could be sprayed. Point E therefore violates Goal 3. We do not wish to achieve goal 4 at the expense of goals, 3, 2, or 1.

The optimum solution point is therefore point F and yields an acreage use of:

70 acres for chaining

20 acres for spraying

The total profit to the rancher would be $205.00. There exists a 25 acre underachievement in goal 3 and the utilization of bottomland acreage is 10 acres. This solution does achieve the stated goals as closely as possible according to the stated priorities and within the restrictions imposed by the constraints.

SIMPLEX SOLUTION

The simplex algorithm for goal programming is similar to the simplex solution of linear programming problems; however, there are several distinct differences.

First, in goal programming, the purpose of the objective function is to minimize the total unattained goal levels. This is accomplished by minimizing the deviational variables through the use of certain preemptive priority factors and differential weights. There is no profit maximization or cost minimization per se in the objective function, however, a cost function may be included as a goal. The preemptive factors and differential weights take the place of the C_j values in linear programming.

Second, the objective function is expressed by assigning priority factors to certain deviational variables. These preemptive priority factors are

multidimensional because priority assignments of different rank may assume any dimensions necessary as long as the dimensions within the same priority levels are homogeneous. It is very important to understand that these priority factors are ordinal rather than ratio values and therefore must take on non-parametric characteristics. This fact necessitates a change in simplex algorithm $(Z_j - C_2)$ from a single row to a matrix of m x n size where m represents the number of preemptive priority levels and n is the number of variables including both decision and deviational variables.

Third, as a result of expressing the simplex criterion in a matrix, a new procedure to identify the optimum column must be used. The relationship between the preemptive priority factors is $P_j >>> P_{j+1}$, which means that P_j always takes priority over P_{j+1}. Based on this, the selection of the optimum column must consider the level of priorities.

The rancher's problem as formulated and solved in the previous pages was:

$$
\begin{array}{llll}
\text{Minimize} & Z = P_1d_1^- + P_2d_4^+ + SP_3d_2^- + 3P_3d_3^- + P_4d_1^+ & & \text{Objective Functio} \\
\text{Subject to:} & X_1 + X_2 + d_1^- \qquad\qquad - d_1^+ & = 80 & \text{A goal} \\
& X_1 \qquad\quad + d_2^- & = 70 & \text{B goal} \\
& \quad X_2 \qquad\quad + d_3^- & = 45 & \text{C goal} \\
& X_1 + X_2 \qquad\qquad + d_4^- - d_4^+ & = 90 & \text{D goal} \\
& X_1, X_2, d_1^-, d_2^-, d_3^-, d_4^-, d_1^+ \geq 0 & &
\end{array}
\tag{9}
$$

Goal D above can also be written as formulated for ease of explanation as shown below:

$$d_1^+ + d_{11}^- - d_{11}^- = 10 \tag{10}$$

where

$d_1^+ \equiv$ overutilization of bottomland

$d_{11}^- \equiv$ difference between the actual utilization of the bottomland acreage and the acres of bottomland utilized

$d_{11}^+ \equiv$ overutilization of the bottomland acreage in excess of 10 acres.

In this formulation, Goal D is only involved with the utilization of the bottomland acreage and Goal A controls the browse acres.

If Goal D (10) was used, the second objective function should read $P_2 d_{11}^+$.

Table 5 outlines the initial tableau for this goal programming problem. The assumptions required for the formulation of a goal programming problem are identical to those required for the conventional linear programming problem. We assume the initial solution is at the origin, and as such, the values of the decision variables are zero. In the first constraint the total utilization of acreage is zero, because:

$$X_1 = X_2 = 0 \tag{11}$$

If X_1 and X_2 both equal zero, then it follows that the utilization of the bottomland acreage equals zero: $(d_1^+ = 0)$. Carrying the logic on further, if $d_1^+ = 0$ then d_1^- which is the underutilization of the browse acreage will be 30 acres: $(d_1^- = 80)$.

Table 5. Initial tableau.

C_j					P_1	$5P_3$	$3P_3$		P_4	P_2
	BASIS	RHS	X_1	X_2	d_1^-	d_2^-	d_3^-	d_{11}^-	d_1^-	d_{11}^+
P_1	d_1^-	80	1	1	1	0	0	0	-1	0
$5P_3$	d_2^-	70	1	0	0	1	0	0	0	0
$3P_3$	d_3^-	45	0	1	0	0	1	0	0	0
	d_{11}^-	10	0	0	0	0	0	1	1	-1
$(Z_2 - C_j)$	P_4	0	0	0	0	0	0	0	-1	0
	P_3	485	5	3	0	0	0	0	0	0
	P_2	0	0	0	0	0	0	0	0	-1
	P_1	80	1	1	0	0	0	0	-1	0

The variable d_1^- is entered in the solution basis and the value 80 is placed in the right-hand-side column. Variable d_2^- and d_3^- are also in the solution basis. We are at the origin so the rancher is not treating any acreage; therefore, d_1^+ is zero. No bottomland is presently utilized, therefore, (d_{11}^+) must also be zero. If d_{11}^+ (overutilization) = 0, then the underutilization d_{11}^- of the bottomland acreage must equal 10. The negative deviational variables (d_i^-) will always appear as a diagonal identity matrix in the solution basis of the initial goal programming tableau.

Now examine the C_j row, Table 5. This row is represented by the preemptive priority factors and the differential weights in the goal programming objective function. The values are the weighted coefficients of the priority ranking assigned to each deviational variable to be minimized.

The simplex criterion $(Z_j - C_j)$ is a 4 x 8 matrix because we have 4 preemptive priority factors and 8 variables, of which 2 are real and 6 are deviational, $4d_i^-$, $2d_i^+$. The most important goal must be achieved to the fullest possible extent before the next-order goal is considered, and so forth. The selection of the entering column is based on the per unit contribution rate of each variable in achieving the most important goal. When the first goal has been completely obtained, then the selection criteria will be based on the achievement rate for the second goal, and so on.

Goal programming problems are always minimization problems. In comparison, the constants in the right-hand-side column of linear programming minimization model represents the total cost of the solution in terms of money and resources; however, in goal programming these same values ($P_4 = 0$, $P_3 = 485$, $P_2 = 0$, $P_1 = 80$) in the right-hand-side column represent the unattained portion of each goal. In this example the second and fourth goals are already completely obtained. The second goal is to minimize the utilization of the ranch's bottomland in excess of 10 acres, and the fourth goal is to minimize the total overutilization of browse land. These goals are met because no acreage is being utilized. The underachievement of the first goal is 80 because the underutilization of the browse acreage is 80 acres. The third goal is to chain as close to 70 acres and spray as close to 45 acres as possible of the mountain land.

The 485 unit value shown in the right-hand side column for the P_3 row of the $Z_2 - C_j$ matrix is computed by multiplying the differential weight for the P_3 values shown in the C_2 matrix by the right-hand side value for the deviational variables being considered ((70 X 5) + (45 X 3) = 485).

<u>Calculation of</u> $Z_j - C_j$

The C_j values represent the priority factors assigned to deviational variables and Z_j values are products of the sum of C_j times constants or coefficients of the A matrix. The Z_j value in the X_1 column will be $P_1 \times 1 + 5P_3 \times 1$ or $P_1 + 5P_3$. The C_j value in the X_1 column is zero as shown in Table 5. Therefore, $Z_j - C_j$ for the X_1 column is $P_1 + 5P_3$. P_1 and P_3 are not non-commensurable so we must list them separately in the P_1 and P_3 rows in the simplex criterion $(Z_j - C_j)$. The $Z_j - C_j$ value will be a one in the P_1 row and a five in the P_3 row in the X_1 column.

The $Z_j - C_j$ coefficient for X_2 column is derived as follows:

$$(P_1 \times 1) + (3P_3 \times 1) \text{ or } P_1 + 3P_3 \tag{12}$$

where a one is entered in the P_1 row of column X_2 and three is entered in the P_3 row of column X_2 in the $Z_j - C_j$ matrix shown in Table 5. The C_j value is again zero.

The $Z_j - C_j$ values for d_1^-, d_2^- and d_3^- will be zero since Z_j values are identical to the respective C_j values. The d_{11}^- column will have a $Z_j - C_j$ value of zero because both the Z_j and C_j values are zero.

The d_1^+ column value is calculated as follows:

$$Z_j = (P_1)\ (-1) + (0)\ (1) = -P_1 \tag{13}$$

Since the C_j value of the column is P_4, $Z_j - C_j$ will be $-P_1 - P_4$. Therefore, -1 is listed in row P_1 and also in row P_3 in column d_1^+.

For the last column the Z_j value = 0 but it is assigned a priority of P_2; therefore, $Z_j - C_j = -P_2$. The -1 coefficient is placed in the P_2 row in column d_{11}^+. This completes the calculation of the $Z_j - C_j$ matrix for the initial (origin) solution.

Selection of Optimum Column and Key Row

The criterion used to determine the optimum column is the rate of contribution of each variable in achieving the most important goal (P_i). Therefore, look for the column with the largest positive value at the P_1 level in the $Z_j - C_j$ matrix and select it as the optimum column. In the example outlined in Table 5, both X_1 and X_2 have the same value; so to break the tie, check the next lowest priority. There is a greater value in X_1 at the P_3 level than X_2, so X_1 is selected as the entering column. The key row is the row which has the minimum value when we divide the right-hand-side values by the coefficients in the optimum column:

Row: P_1	80/1	= 80	
$5P_3$	70/1	= 70 enter 70 units of X_1 exit d_2^-	
$3P_3$	45/0	= undefined	(14)
	10/0	= undefined	

By entering X_1 into the basis the underutilization of the regular range acreage and the underachievement of the utilization goal for acres of chaining will be affected because of the coefficients existing in the d_1^- and d_2^- rows.

Row $5P_3$ is the most constraining constraint and therefore, becomes the tool row. The tool row is used to reduce the elements of the entering column to a column vector of zeros and a one. The one will be in the row of the variable which is entering the basis.

The following calculations are for only one iteration, but the same process is conducted until optimality is established.

d_2^- =	70	1	0	0	1	0	0	0	0	New Row 2*
Row d_1^- =	80	1	1	1	0	0	0	-1	0	
(Row d_2^-) (-1) =	-70	-1	0	0	-1	0	0	0	0	
	10	0	1	1	-1	0	0	-1	0	New Row 1*
Row d_3^- =	45	0	1	0	0	1	0	0	0	New Row 3*
Row d_{11}^- =	10	0	0	0	0	0	1	1	-1	New Row 4*
Row P_3 =	485	5	3	0	0	0	0	0	0	
(Row d_2^-) (-5) =	-350	-5	0	0	-5	0	0	0	0	
	135	0	3	0	-5	0	0	0	0	New $Z_j - C_j$ Row 2*
Row P_1 =	80	1	1	0	0	0	0	-1	0	
(Row d_2^-) (-1) =	-70	-1	0	0	-1	0	0	0	0	
	10	0	1	0	-1	0	0	-1	0	New $Z_j - C_j$ Row 4*

The $Z_j - C_j$ rows 1 and 3 are the same as those in Table 5.

* are the new rows for Table 6.

Table 6. First iteration.

C_j					P_1	$5P_3$	$3P_3$		P_4	P_2
	BASIS	RHS	X_1	X_2	d_1^-	d_2^-	d_3^-	d_{11}^-	d_1^+	d_{11}^+
P_1	d_1^-	10	0	1	1	-1	0	0	-1	0
	X_1	70	1	0	0	1	0	0	0	0
$3P_3$	d_3^-	45	0	1	0	0	1	0	0	0
	d_{11}^-	10	0	0	0	0	0	1	1	-1
	P_4	0	0	0	0	0	0	0	-1	0
	P_3	135	0	3	0	-5	0	0	0	0
	P_2	0	0	0	0	0	0	0	0	-1
	P_1	10	0	1	0	-1	0	0	-1	0

Examine Table 6. The Z_j values in the constant column:

$$\begin{aligned} P_4 &= 0 \\ P_3 &= 135 \\ P_2 &= 0 \\ P_1 &= 10 \end{aligned} \tag{15}$$

indicate the unattained portion of the first goal has decreased considerably, 70 to be exact. This is as expected because the value of Z_j should decrease at each step while approaching an optimum point.

Examining the P_1 row, a positive one in column X_2 indicates we can further minimize the deviations from the first goal by entering X_2.

Now, to define the entering column and key row examine Table 6. We are still concerned with minimizing P_1 to zero. X_2 has the only positive coefficient in the P_1 row; therefore, enter X_2. Now select the key row:

$$\begin{array}{lll} \text{Row:} & d_1^- & 10/1 = 10 \text{ enter 10 units of } X_2 \text{ exit } d_1^- \\ & x_1 & 20/0 = \text{undefined} \\ & d_3^- & 45/1 = 45 \\ z & d_{11}^- & 10/0 = \text{undefined} \end{array} \qquad (16)$$

Row d_1^-, Table 6, becomes the new tool row. By calculations similar to these in the first iteration, we arrive at Table 7, which has an X_2 column vector with a one in row X_2, showing this variable is in solution because all other values in this column vector are zero.

Table 7. Second iteration.

C_j					P_1	$5P_3$	$3P_3$		P_4	P_2
	BASIS	RHS	X_1	X_2	d_1^-	d_2^-	d_3^-	d_{11}^-	d_1^+	d_{11}^+
	X_2	10	0	1	1	-1	0	0	-1	0
	X_1	70	1	0	0	1	0	0	0	0
$3P_3$	d_3^-	35	0	0	-1	1	1	0	1	0
	d_{11}^-	10	0	0	0	0	0	1	1	-1
	P_4	0	0	0	0	0	0	0	-1	0
	P_3	105	0	0	-3	-2	0	0	3	0
	P_2	0	0	0	0	0	0	0	0	-1
	P_1	0	0	0	-1	0	0	0	0	0

Table 7 shows that the best way to achieve the most important goal completely is by spraying 10 acres and chaining 70 acres of mountain land. This combination is sufficient to achieve the first, second, and fourth goals. The third goal is not completely attained since the utilization of available acreage is 35 acres short of complete attainment; d_3^- of 35 can be seen in Table 7. At this point, there is no further attainment required at the P_1 and P_2 levels because all coefficients shown in the $Z_j - C_j$ matrix at these priority levels in Table 8 are either zero or negative.

Selection of the entering column and key row are then determined on the basis of level P_3. We are now concerned with the minimization of P_3; therefore, by Table 7, column d_1^+ has the only positive coefficient in row P_3. Select d_1^+ as the entering column. Now select the most limiting constraint row.

$$\begin{aligned} \text{Row } X_2 &= 10/-1 = \text{Negative} \\ X_1 &\quad 70/0 = \text{Undefined} \\ d_3^- &\quad 35/1 = 35 \\ d_{11}^- &\quad 10/1 = 10 \end{aligned} \tag{17}$$

Row d_{11}^- is found to be the most constraining constraint and will be used as the tool row. The one in column d_{11}^- and row d_4^- becomes the print element. The calculations, once completed, yield Table 8.

Table 8. Third iteration (optimum solution).

C_j					P_1	$5P_3$	$3P_3$		P_4	P_2
	BASIS	RHS	X_1	X_2	d_1^-	d_2^-	d_3^-	d_{11}^-	d_1^+	d_{11}^+
	X_2	20	0	1	1	-1	0	1	0	-1
	X_1	70	1	0	0	1	0	0	0	0
$3P_3$	d_3^-	25	0	0	-1	1	1	-1	0	1
P_4	d_1^+	10	0	0	0	0	0	1	1	-1
	P_4	10	0	0	0	0	0	1	0	-1
	P_3	75	0	0	-3	-2	0	-3	0	3
	P_2	0	0	0	0	0	0	0	0	-1
	P_1	0	0	0	-1	0	0	0	0	0

The third goal is not completely attained as shown by the positive value of three in the $Z_j - C_j$ matrix at the P_3 level column d_{11}^+. Noting the positive three in the d_{11}^+ columns, we could introduce d_{11}^+ in an attempt to further attain the third goal, but because of the (-1) in the level P_2 the additional

achievement of the third goal would be at the expense of the complete achievement of the second goal. The same logic also applies to the one in column d_{11}^- at the level P_4. The rule is that if there is a positive element at a lower priority level in the $Z_j - C_j$ matrix, the variable in that column cannot be introduced into the solution if there is a negative element at a higher priority level (lower numerical value).

Table 8 outlines the optimum goal programming solution. This solution enables the decision maker to attain his goals as closely as possible within the established decision environment and the hierarchial priority structure. To decrease the underachievement of level P_3 from a Z_j value of 105 to 75 we sacrificed the complete attainment of the fourth goal by 10 units as shown at the P_4 level.

The optimum solution is:

$$\begin{aligned} X_1 &= 70 \\ X_2 &= 20 \\ d_1^+ &= 10 \\ d_3^- &= 25 \end{aligned} \tag{18}$$

The rancher should, therefore, treat 70 acres by chaining and 20 acres by spraying with 10 acres of bottomland utilized. This results in a 25 acre underachievement of the goal establishing a maximum of 45 acres for spraying. This level of management enables the rancher to attain the two most important goals completely, and the next two goals as completely as possible under the given constraining system.

Table 8 points out the areas of conflict. Conflict between goals can be pinpointed through review of the $Z_j - C_j$ matrix. Conflict exists between the second and third goals in column d_{11}^{+}, and between the third and fourth goals in column d_{11}^{-}. Knowledge of this type enables the decision maker to rearrange the priority structure if the unachieved goals are assumed more important than originally believed. The decision maker therefore, has the opportunity to evaluate the soundness of his priority structure for his goals. Analysis of the coefficients in the A and B matrices gives the decision maker the capability of identifying the exact trade-offs between goals.

The goal programming formulation and the linear programming formulation both generated identical results. The conventional linear formulation treated the first two goals as constraints and then maximized profit. This does not mean that linear programming would yield the identical answer if we converted some of the management goals to constraints. It could be quite possible that none of the management goals in public land management would involve profit maximization or cost minimization. The standard linear programming formulation might very well result in an infeasible solution.

Steps of the Simplex Method of Goal Programming:

(i) set up the initial tableau from the goal programming models,

(ii) calculate initial $Z_j - C_j$ matrix,

(iii) determine the new entering variable,

(iv) determine the existing variable from the solution base,

(v) determine the new basic feasible solution, and

(vi) determine whether the solution is optimal.

EXAMPLE 2

Area Description

This simple example was developed from data collected on the Pawnee National Grassland which is located in northeastern Colorado. The example is constructed in order to demonstrate that goal programming is more flexible than conventional linear programming.

The 51,292 acre study area was divided into three separate range response units. These units were defined by similarities in soil characteristics, vegetational structure and predicted response to a number of ecosystem manipulations.

Range type I is 17,866 acres of predominantly native short-grass vegetative species of which blue grama (*Bouteloua gracilis*) and buffalograss (*Buchloe dactyloides*) are dominant. The abundance of these two species makes this unit a warm season range. Production during the spring on this type of range is generally quite low ranging from 125 lbs/acre to 225 lbs/acre with an average production of 150 lbs/acre. If a blue grama range is grazed fairly heavily during the late spring months of May and June, the production for the remainder of the year will be reduced about 30 to 40%. The blue grama range type is capable of withstanding very heavy grazing if grazing is started during the later portion of summer and into fall. The range forage can be grazed down to 300 lbs/acre or less without affecting the following years production if grazing is started during the late summer season. Carbohydrate reserves are stored in the plant roots for the next year's growth. Removal of aboveground plant material will subsequently have no effect on the carbohydrate reserves of the plants at this time.

Range type II is composed mainly of western wheatgrass (*Agropyron smithii*) and crested wheatgrass (*Agropyron cristatum*). This range type is 18,205 acres of the mid-grass type. The standing crop in the spring can range from 0-450 lbs/acre depending on the amount of precipitation during the early spring months of March and April. The average standing crop on this range type is 350 lbs/acre during the spring. During summer, standing crop average is around 600 lbs/acre while in the fall of the year there may be 700 lbs/acre and during winter there will be less than 675 lbs/acre of available forage for livestock and wildlife. If the wheatgrass is available in sufficient quantity, it is best to utilize this range type during late spring. Very good forage is available for late fall and winter use. It is best, however, to graze wheatgrasses during late spring if at all possible. Range type II is found primarily in the heavier textured soils of the Pawnee National Grassland.

The 15,221 acres of range type III is composed predominantly of shrubs and sub-shrubs. The main species found in this range type include saltbush (*Atriplex canescans*) and sagebrush (*Artemisia* spp.). This range type is of little value for cattle production but offers a beneficial habitat for the native wildlife in the area. If saltbush is grazed during the spring months there will be no regrowth for the remainder of the year; however, normal growth will resume the following spring. Forage standing crop during the spring may range from 0-500 lbs/acre; here again, forage standing crop is dependent upon the time and amount of precipitation. The average standing crop of range type III in spring is around 400 lbs/acre. An average summer standing crop of 1800 lbs/acre can be expected on this range

type while in late fall available forage will decrease by almost half to 950 lbs/acre. During winter, 50% of the available fall forage can be expected available for animal use. Range type III is located predominantly in the flood plains or overflow sites.

Requirements for the fixed resource base are dictated by numerous user groups. The major human user groups include:

(i) Livestock Producers

This study area has 52 individual rancher permittees who require a year-round grazing system for at least 24,112 animal unit months (AUM's). The permittees further stipulate that no more than 10% or 2,411 steer AUM's can be grazed; the remainder must be cow/calf units. All domestic grazing is subject to the restrictions outlined by the Forest Service regulations and Grazing Association By-laws; Forest Service Regulations and Grazing Association By-laws were both used to constrain the model. The listing of these two user requirements seems worthy of the effort to further enlighten the reader.

a. Forest Service Regulations

(1) commensurability

(2) transfer of permit

(3) season of use

b. Association By-laws

(1) Forest Service responsibilities

(2) Association responsibilities

(3) Fire Control responsibilities

(ii) Recreational Users

Recreation plays an important role on the Pawnee National Grassland. In particular, camping and other recreational facilities in the park near Briggsdale, as well as hunting and bird watching, support over 28,200 user days.

The camping facilities near Briggsdale, Colorado, are included in a forty-acre tract of land. This campground includes picnic areas, ball diamond, shelter house, and, of course, toilets. Management stipulates that no more than two campgrounds can be built; 28,200 user days were accommodated last year. The number of user days varies with the season; summer, 9306; fall, 6204; and winter, 12,690. The budget used for maintenance, improvements and new developments consists of $2700 per year per campground.

Hunting also takes place on the National Grasslands site. This recreational sport is directed mainly toward the population of antelope and a small herd of deer. Hunting of smaller game such as rabbits and other small rodents in conjunction with predators such as the coyote constitutes the bulk of this recreational activity in the area.

This simplified example considered the use of the study area by 50 head of deer and 1300 head of antelope on a year-round basis.

Bird watching is favored for the area as pointed out by trips made to the area by the National Audubon Society. Several thousand species of birds may be found on the National Grasslands ranging from eagles to waterfowl.

Human users, however, do not comprise the complete list of resource users; in addition, any decision-making process must consider other users and constraints as dictated by the ecosystem.

LINEAR PROGRAMMING MODEL

The model formulation included 18 resource and user constraints and 25 decision variables of which 12 were management alternatives (resource producers) and 13 products (resource users). This model will later be modified and converted to a goal programming (G.P.) formulation.

Resource and user constraints attempt to limit the decision environment to the resource base and capabilities that presently exist on the study area. Each constraint is assumed to be linear and is composed of the same units of measure, i.e., tons, pounds, user days, etc. The constraints used in this simple model are listed in Table 9. Fig. 5 shows the general format for the goal programming formulation being used.

Constraint Explanation:

(i) Constraints 1, 2, and 3, outline the fixed resources for this problem and are considered less than or equal to constraints because not all acreage need be used.

Constraint 1 - Acres of range type I must be less than 17,866 acres.

Constraint 2 - Acres of range type II must be less than 18,205 acres.

Constraint 3 - Acres of range type III must be less than 15,221 acres.

(ii) Constraints 4 and 5 are general system constraints as outlined below.

Constraint 4 - Campground Development - The development of 3-acre campsites - we require no more than 2 campsites be developed.

Constraint 5 - Steer Animal Unit Months - The ranchers desire no more than 2,411 steer units.

(iii) Constraints 6 through 14 outline the variable or flow resource constraints.

Table 9. Linear programming constraints.

Constraint No.	Constraint Heading	Type of Equality	R.H.S. Coefficient*
1	Acres Range Type I	$\leq$	17,866.
2	Acres Range Type II	$\leq$	18,205.
3	Acres Range Type III	$\leq$	15,221.
4	Campground Development	$\leq$	2.
5	Steer Animal Unit Months	$\leq$	2,411.
	Dry Matter for Domestic Use		
6	Season 1	$\geq$	0.
7	Season 2	$\geq$	0.
8	Season 3	$\geq$	0.
	Dry Matter for Wildlife Use		
9	Season 1	$\geq$	0.
10	Season 2	$\geq$	0.
11	Season 3	$\geq$	0.
	Recreation User Days		
12	Season 1	$\geq$	0.
13	Season 2	$\geq$	0.
14	Season 3	$\geq$	0.
15	Total User Days	$\geq$	28,200.
16	Cow-calf Animal Unit Months	$\geq$	21,701.
17	Deer Unit Years	$\geq$	50.
18	Antelope Unit Years	$\geq$	1,300.

* R.H.S. stands for right-hand-side.

GOAL PROGRAMMING FORMULATION

MANAGEMENT SCHEMES	PRODUCTS (USERS)	DEVIATIONAL VARIABLES		
PRODUCTION RATE FOR GOAL CONSTRAINTS OF VARIABLE RESOURCES	USE RATES FOR GOAL CONSTRAINTS OF VARIABLE RESOURCES	"LINK" MATRIX BETWEEN DEVIATIONAL VARIABLES AND GOAL CONSTRAINTS	= ≥ ≤	GOAL LEVELS
"LINK" MATRIX BETWEEN PRODUCTION OF "FACTORS" ON FIXED RESOURCES & MANAGEMENT SCHEMES	NULL SUBMATRIX	NULL SUBMATRIX	≤	FIXED RESOURCES (LAND, SUPPLEMENT)
PRODUCTION RATES OF VARIABLE RESOURCES	USE RATES OF VARIABLE RESOURCES	NULL SUBMATRIX	≥	LOWER LIMIT OF VARIABLE RESOURCES
NULL SUBMATRIX	"LINK MATRIX" BETWEEN USERS & REQUIREMENTS	NULL SUBMATRIX	≥	QUANTITY OF USER (PRODUCT) REQUIREMENT
		OBJECTIVE FUNCTION OF WEIGHTS AND PRIORITY FACTORS FOR THE MINIMIZATION OF THE DEVIATION FROM EACH GOAL		

Fig. 5. Goal programming model formulation.

Constraints 6, 7, and 8 outline the dry matter available for domestic cattle use.

Constraint 6 - Forage available in pounds per acre during season 1. Non-negativity requires right-hand-side to be greater than zero.

Constraint 7 - Forage available in pounds per acre during season 2. Right-hand-side must be greater than zero.

Constraint 8 - Forage available in pounds per acre during season 3. Right-hand-side must be greater than zero.

Constraints 9, 10, and 11 outline the dry matter available for wildlife consumption.

Constraint 9 - Wildlife forage available in pounds per acre during season 1. The right-hand-side must be greater than or equal to zero.

Constraint 10 - Wildlife forage available in pounds per acre during season 2. The right-hand-side must again be greater than or equal to zero.

Constraint 11 - Wildlife forage available in pounds per acre during season 3. The right-hand-side again assumes a value of zero.

Constraints 12, 13, and 14 outline the recreation user days constraints. The dimensions of these constraints become user days.

Constraint 12 - User days during season 1 must be greater than or equal to zero.

Constraint 13 - User days during season 2 must be greater than or equal to zero.

Constraint 14 - user days during season 3 must be greater than or equal to zero.

(iv) Constraints 15 through 18 establish the lower limits of product or resource user groups.

Constraint 15 - is in dimensions of user days which must be greater than 28,200 user days per year.

Constraint 16 - cow-calf AUM's are the dimensions of this constraint and must be greater than or equal to 21,701 animal unit months.

Constraint 17 - Deer unit years become the dimension of this constraint. We require at least 50 deer be supported per year on this acreage.

Constraint 18 - Antelope unit years is the dimension to which this constraint must conform. This area must support at least 1300 antelope for one year.

Alternative Explanation:

As explained above, three range types were delineated in the area and used as the fixed resource base for the model. Four management alternatives were used on each range type. In this simple example we were concerned with the allocation of domestic cattle grazing among the three range types. The four management alternatives for each range type were defined as follows:

1. As is grazing during the period from May 15 to August 15. This alternative advocates using the area as it is now with no work done to improve production, utilization or quality of available forage. The cost of this alternative is based on two figures; the grazing fee paid by the rancher to the Forest Service is $1.34/AUM; and secondly, the present management plan is based on the fact that it takes 4.3 acres to yield one AUM. Using these two figures, a production cost of $0.31/acre is derived. When this alternative is selected it indicates that revenue is high enough to make it economically infeasible to do any major range improvement work.

2. As is grazing during the period from August 16 to September 30. Other than the dates, the alternative and cost are the same as in number 1.

3. As is grazing during the period from October 1 to May 14. Again, this alternative is defined the same as in number 1.

4. This alternative defines campground development. The size and capacity and cost of the campground will vary depending upon the range type being considered.

Decision Variable Explanation:

The decision variables used in this formulation are listed in Table 10. The matrix is constructed so that the 18 constraints form the rows of the model while the variables form the columns. Columns are related to rows (variables to constraints) via resource production or use coefficients. These coefficients are generally averages per unit considered.

Table 10. Decision variables.

Variable X_1 through X_{12} define the resource producing alternatives for this model.

Range Type I	Range Type II	Range Type III
X_1 as is use season 1	X_5 as is use season 1	X_9 as is use season 1
X_2 2	X_6 2	X_{10} 2
X_3 3	X_7 3	X_{11} 3
X_4 campground development	X_8 campground development	X_{12} campground development

Variables X_{13} through X_{25} define the resource users (products) for this formulation.

Graze Cow-Calf Animal Unit Months

X_{13} season 3 - October 1 to May 14

X_{14} year round

X_{15} season 2 - August 16 to September 30

X_{16} season 1 and 3 - May 15 to August 15, and October 1 to May 14

Graze Steer Animal Unit Months

X_{17} year round

X_{18} season 2 - August 16 to September 30

X_{19} season 3 - October 1 to May 14

X_{20} season 1 and 3 - May 15 to August 15, and October 1 to May 14

X_{21} deer unit years

X_{22} antelope unit years

Recreation User Days

X_{23} season 1

X_{24} season 2

X_{25} season 3

Dry Matter Coefficients

Matrix coefficients for dry matter constraints were obtained by consideration of the length of that season in days, the growing season involved, and the average production per acre estimates pertaining to those seasons. All dry matter matrix coefficients are expressed in pounds of available air dry forage per acre.

During a personal interview with the manager of the area, it was understood that current management required approximately 300 pounds of air dry forage per acre be left standing at the end of the grazing season. All range types must meet this criterion; therefore, this amount was deducted from all coefficients entered in the matrix.

User Requirements

Cow-Calf and Steers: Matrix coefficients relate the amount of forage required for maintenance by the grazing unit. These coefficients are expressed in pounds of forage per animal unit month and consider the length of time for that segment.

Variance in coefficients can be attributed to animal weight, maintenance requirements for the different seasons, and different classes of animals.

Deer and Antelope: The daily maintenance requirements for deer are entered as pounds of forage per animal unit per year (AUY). Consumption was then divided among the seasons and, therefore, vary according to length of season and requirement of the animals during that time of year.

To show competition for the limited forage resource, we divided the total forage production into domestic and wildlife forage. Realizing there is no clear division, we assumed that deer competed for 25% of the available

domestic animal forage and antelope competed for 40% of the forage. The model, therefore, shows these animals consuming both types of forage.

Recreation

Recreational use and its associated impact creates a problem when some quantification is necessary. How do you assign a value to a user day? Many people have tried to assign a value base on a monetary value or a utility function. A sounder approach would be to derive a form of opportunity cost. An acre used by a recreational activity usually decreases its potential for another use, say grazing or domestic forage utilization. Assuming that a recreational carrying capacity can be calculated and an average biomass production per acre can be estimated, a figure representing forage consumption per user day can be calculated.

$$\frac{\text{Pounds of Forage/Acre}}{\text{User Days/Acre}} = \frac{\text{Pounds}}{\text{User Days}}$$

The resulting consumption value can be seen under the user days columns X_{23} - X_{25} in the model. This assumption is valid if the user considers recreational use and grazing use as competitive activities. In this case, they are competing for acreage and forage.

Objective Function Values

The coefficients in the objective function for the management alternatives represents the cost per unit activity. This is usually, but not always on a per acre basis. The objective function values for the products represent the current sale price. Worked into this price are several factors: initial price of the heifer; keeping her for 8 years; sale price of the calves she gives (90% calf crop); canner sale price. It was assumed that the steers

were born on the ranch, kept for one year and sold. Therefore, the prices stated in the model are current sale prices for steers as quoted by the Fort Collins Livestock Auction.

Several other use alternatives were then added for consideration. Deer and antelope exist on the area in limited numbers at the present time, and do compete for the fixed land resource base. No accurate value could be determined for a benefit so the objective function was left empty.

RESULTS

In this section the results from two parametric runs of the linear programming model are discussed. Goals will then be formulated and the L.P. model will be changed to a goal programming (G.P.) formulation. The G.P. formulation was then run twice using the same constraint formulations as used in the two parametric L.P. runs. Results of the L.P. and the G.P. formulations will then be compared.

Parametric Run 1 (Linear Programming)

The parametric runs completed for this exercise only varied in the right-hand-side values of the constraints. Parametric run 1 included the constraint values shown in Table 11, using these values.

In practice, it is up to the manager to set these right-hand-side values. The values changed represent either increased or decreased goals arrived at through the users' evaluation of the previous run's results. As an example, let us assume we were satisfied with the results of the first run, except we would like to see the effect on the system of increased recreation use. These changes can be seen in Table 11, constraints 12, 13, and 14.

Table 11. Right-hand-side value for the first parametric run.

Constraint Number	Right-Hand-Side Headings	Type of Equality	Right-Hand-Side Values
1	acres range type 1	$\leq$	17866
2	acres range type 2	$\leq$	18205
3	acres range type 3	$\leq$	15221
4	number of campgrounds	$\leq$	2
5	steer animal unit months	$\leq$	2411
6	domestic livestock forage-season 1	$\geq$	0
7	domestic livestock forage-season 2	$\geq$	0
8	domestic livestock forage-season 3	$\geq$	0
9	wildlife forage-season 1	$\geq$	0
10	wildlife forage-season 2	$\geq$	0
11	wildlife forage-season 3	$\geq$	0
12	user days-season 1	$\geq$	9306
13	user days-season 2	$\geq$	6204
14	user days-season 3	$\geq$	12690
15	total allowable user days	$\geq$	28200
16	cow-calf animal unit months	$\geq$	21701
17	deer unit years	$\geq$	50
18	antelope unit years	$\geq$	1300

Using these R.H.S. values, an optimal solution was obtained which would maximize net revenue within the constraint requirements of the formulation. This solution would generate $1,515,184 of profit or an average of $29.54/acre/year.

The management plan dictated the following actions. We should use 913 acres of range type I during season 2 and 1694 acres will be grazed during season 3. Campgrounds in range type I will occupy 7 acres, bringing the total acreage in range type I to 17,866 acres, the total available. The ranchers should use 18,205 acres of range type II during season 2. The range type III acreage is to be utilized during season 1 and season 3, 1,641 and 13,574 acres, respectively. The campgrounds in range type III are to take up to 6 acres using the maximum allowable acres, 15,221.

The ranchers are to graze 3357 cow-calf units during season 3, and 2222 cow-calf units during season 2. Steers are to number 804 animal unit months which will be grazed during seasons 1 and 3.

This solution also allows for 50 head of deer and 1300 head of antelope on a year-round basis. In addition, the solution provides for 16,354 user days during season 1, and 11,844 user days during season 2.

Parametric Run 2

Let's assume review of the first parametric run causes us to change the constraints shown in Table 12. The right-hand-side requirement for all other constraints remained the same as those shown in Table 11.

Table 12. Changes in right-hand-side values for parametric run 2.

Constraint Number	Type of Equality	Right-Hand-Side Values
16	$\geq$	43402
17	$\geq$	400
18	$\geq$	10400

For demonstration purposes this second parametric L.P. run was formulated to yield an infeasible solution to show the increased flexibility of goal programming by deriving an optimal solution to a formulation which proved infeasible under normal linear programming procedures.

Infeasibility was accomplished by considering forage production to be limited and raising the right-hand-side requirement for:

cow-calf units from 21,701 AUM's to 43,402 AUM's

deer units from 50 AUY's to 400 AUY's

antelope units from 1,300 AUY's to 10400 AUY's

Goal Programming Results

The main difference between L.P. and G.P. is that G.P. has multiple objective function while L.P. only has the single objective function. The goal programming formulation was identical to that of the L.P. except for a deviational variable matrix.

Let's assume the Crow Valley Association has a meeting and they formulate the following goals:

1. They desire the association to make as close to $1,515,184 as possible. This value was determined by using the optimum profit solution generated by the first parametric L.P. run.

2. Realizing that forage is limited, the ranchers desire to limit the over-attainment of deer to under 50 AUY's and the antelope to under 1300 AUY's.

3. Goal 3 involves domestic cattle animal unit months. They should not be less than 21,701; therefore, we must minimize the underachievement.

4. Steer units should be less than 2,411 AUM's. Therefore, minimize the overachievement. We would rather have achieved goal 3 than 4.

5. The ranchers would like to see user days in season 1 be greater than 9306; therefore, minimize the underachievement of this goal.

6. They would like to have the user days in season 2 greater than 6204; therefore, minimize the underachievement of this goal.

7. They further desire at least 12,690 user days in season 3. Minimize the underachievement.

8. As a further constraint the ranchers need at least 28,200 user days. We, therefore, wish to minimize the underachievement of this goal.

Collectively, the ranchers work to utilize all of the land they have available. Goals 9, 10, and 11 are then formulated.

9. Minimize the underutilization of the type 1 acres.

10. Minimize the underutilization of the type 2 acres.

11. Minimize the underutilization of the type 3 acres.

To obtain a solution via goal programming, the ordinal ranks of the goals are needed. This enables the most desired goals to be achieved before the lesser goals are even considered. In this way, the final solution represents the optimal combination of conflicting goals, as dictated by the priority structure.

In our example, goal 1 was assigned priority 1, goal 2 is assigned priority 2; goal 3 acquires a value $2P_3$; and goal 4, P_3. Goals 5, 6, 7 and 8 are assigned P_4 priority. We would much rather meet the total user day goal than the seasonal user day goals; goal 8 is therefore, assigned a $2P_4$ value. Goals 9, 10 and 11 are assigned a P_5 priority.

These priorities comprise the ordinal rankings for the goal programming objective function. Deviations from these goals are minimized to generate an optimal solution.

Parametric Run 1 (Goal Programming)

The first run for the parametric goal programming formulation was the same as the first run for the conventional L.P. formulation. Table 13 shows the comparison between the two formulations.

The values obtained by both methods are essentially the same, because all goals were completely achieved and the demands on the resources did not exceed the capability of the ecosystem modeled.

Parametric Run 2

The second parametric goal programming run will show a case of conflicting multiple goals. The same matrix coefficients and constraint rows were used as those in the second L.P. parametric formulation, which proved infeasible.

The solution derived by the second G.P. parametric run differs from that of the first run, as shown in Table 14.

Table 13. Goal programming and linear programming comparison.

X(I)	L.P. Value	G.P. Value	Heading
2	913	913	acres range type I, season 2
3	16,946	16,946	acres range type I, season 3
4	1	1	campgrounds range type I
6	18,205	18,205	acres range type II, season 2
9	1,641	1,641	acres range type III, season 1
11	13,574	13,574	acres range type III, season 3
12	1	1	campgrounds range type III
13	3,357	3,357	cow-calf units, season 3
15	2,222	2,222	cow-calf units, season 2
19	0	0	steer AUM's, season 3
20	804	804	steer AUM's, season 1 and 3
21	50	50	deer unit years
22	1,300	1,300	antelope unit years
23	16,356	16,356	user days type I
24	11,834	11,844	user days type II
Objective function	1,515,185	1,515,185	dollars

Table 14. Comparison between goal programming run 1 and goal programming run 2.

X(1)	First Run	Second Run	Heading
2	913	0	acres range type I, season 2
3	16,946	17,857	acres range type I, season 3
4	1	1	campgrounds type I
6	18,205	11,220	acres range type II, season 2
7	0	6,985	acres range type II, season 3
9	1,641	8,986	acres range type III, season 1
11	13,574	6,229	acres range type III, season 3
12	1	1	campgrounds range type III
13	3,357	0	cow-calf units, season 3
15	2,222	0	cow-calf units, season 2
18	0	450	steer AUM's, season 2
19	0	4,176	steer AUM's, season 3
20	804	0	steer AUM's, season 1 and 3
21	50	0	deer unit years
22	1,300	9,987	antelope unit years
23	16,356	16,635	user days type I
24	11,844	11,844	user days type II
Objective function	1,515,185	1,515,185	dollars

The objective function value remains the same even though some of the values associated with the decision variables have changed; because profit was a goal and, therefore, the value could not exceed or be less than the specific values. This formulation could change easily by relaxing the requirements on the profit values.

Goal programming's extra flexibility allows the above solution. In obtaining this solution and meeting the first priority goal, steer production is favored over cow-calf production. Deer unit years, a priority Z goal, is not met because of lack of available forage. Table 14 shows that the grazing use of each range type during the three seasons has significantly changed. This is an optimal solution only under the present assigned priorities. The solution basis shown in Table 14 for the second run, shows the complete achievement of goals for priority 1, 4, and 5 goals. The underachievement of priority 2 goal was composed of 400 deer unit years and 412 antelope unit years goal. The attainment of priority 3 goal values arose because we had an overachievement of steer units by 34,315 and an underachievement of 43,402 cow-calf units.

The L.P. solution for this run was deemed infeasible showing that L.P. does have a limited solution scope which may not represent reality. The goal programming formulation, however, does arrive at a solution which is optimal under the specified decision environment. While the linear programming formulation has no solution, the goal programming formulation has a solution at the expense of the lower order conflicting goals. G.P., therefore, allows the manager more flexibility in predicting the consequences of a proposed management scheme.

LIMITATIONS AND PROBLEMS

Goal programming eliminates some of the limitations of conventional linear programming but still suffers from the linear programming limitations of proportionality, additivity, divisibility. The formulation usually needs to be deterministic; however, some work has been done with dynamic, parametric and stochastic goal programming. The most important limitations of goal programming lie in the fact that the model simply provides the best solution under the given set of constraints and priority structure. Therefore, if the decision-maker's goal priorities are not in accordance with the organizational objectives, the solution will not be the global optimum for the organization.

INFORMATION REQUIREMENTS

Since we are dealing with the systems concept, it seems fitting to show a systems diagram of the information flows to and from any goal programming model. Information requirement begins with the definition of the problem. Once the problem is defined, information must be collected either through resource inventories or a literature review. This data is then sorted and that which is needed is then incorporated into either the ecological, simulation or economic models. The results from these models aid in the evaluation of the land capabilities and present state of the ecosystem. In addition, simulation models can be used to predict the future outcome given a specific action or activity. These models, therefore, build time into the system's evaluation. Economic models aid in approaching reality

through constraints and goals which are involved in the interaction of the resource user, primarily people and the resource base. These three models are used to define broad quantitative ecological and economic parameters as deemed feasible by the system.

The results from these models pass directly to the manager and the normative model compartments. A form of arbitration then becomes evident because this immense data base must be reduced to an abstracted initial operating system, with its parameters, data goals and goal priority ranking defined. The parameters and data for the system may come from the manager section just as the goals and priority ranking will come from the normative model. Feedback between these two compartments is also inherent because the roles of each compartment may be interchanged.

The arbitrated results become the input into the optimizing, goal attaining, resource allocating goal programming machine.

Results generated would hopefully show the response of a requirement or proposed action dictated by the regional organizational level and indicate if there is a need for reweighting an alternative. Results generated would add further identification of states to the ecological subsystem plus enable observation of the expected perturbation of ecological states as a result of a goal, requirement, or proposed management activity.

The output from this goal machine must then be subjected to another period of arbitration in the evaluating of the results. This evaluation is aided by the non-quantitative constraints generated by the manager and and the normative model. Once the arbitration is completed, if the results are unsatisfactory, the problem must be reformulated by the manager and the normative model compartment, and run back through the system. If the results

are satisfactory the task is to develop and implement the plan of action. This is a dynamic system because the input and output are continually changing over time.

Any results generated are only meaningful if they can continually be updated and change as requirements change; therefore, once the goal programming procedure and basic model are developed there will be a need for the development of a responsive interactive system which would be capable of instantly showing the impact or effect of changing or adding a specific goal requirement. Feedback must, therefore, go from the plan development to the initial data collection and, thus, back through the complete system.

The success of goal programming as applied to natural resource decision processes in this project depends on each and every subsystem and the data they generate.

SUMMARY AND CONCLUSIONS

Natural resource decision makers are concerned with the allocation of scarce natural resource. Assume we have defined n as different input resources that are limited to certain quantities, and there are m different types of outputs that result from various combinations of these resources, the decision problem is to determine the optimum combination of input resources to achieve certain goals so that the total goal attainment can be maximized.

Goal programming has been successfully applied to problems of resource allocation and many marketing situations with multiple goals. It is a relatively new technique, and one in which its true potential is yet to be determined. However, it appears that the potential applicability of goal programming may be as wide as or wider than that of linear programming.

Goal programming has a great deal of flexibility that is lacking in linear programming. Furthermore, the approach of multiple-goal attainment according to established priorities is readily suitable to most management decision problems (Lee, 1972).

The public often views the soundness or rationality of a decision-making process by the degree of organizational goals achieved by a decision. Goal programming measures the degree of goal attainment in a constrained environment. The true value of goal programming, therefore, lies in the ability to solve problems involving multiple, conflicting goals according to a normative priority structure.

Goal programming utilizes the good points of the conventional linear programming techniques while improving on the basic limitation of linear programming, namely the unidimensional objective function. Goal programming can be a useful tool to aid in multi-resource planning.

APPENDIX A

BIBLIOGRAPHY ON GOAL PROGRAMMING

BIBLIOGRAPHY

Bare, Bruce B. 1971. Applications of operations research in forest management: A survey. Quantitative Science Paper No. 26, presented at the American Statistical Association meeting in Fort Collins, Colorado, August 24, 1971. Pages 1-56.

Bartee, Edwin M. 1971. Problem solving with ordinal measurement. Manage. Sci. 17(10). Pages 1-22.

Belenson, Sheldon M., and Kailash C. Kadur. 1973. An algorithm for solving multicriterion linear programming problems with examples. Operational Res. Quart. 24(1):65-77.

Bell, Enoch F. 1972. Programming for multiple use decisions. Virginia Polytechnic Institute and State University, Blacksburg, Virginia. Submitted to U.S. Forest Service, Pacific Northwest Forest and Range Experiment Station, Oregon. 37 pages.

Benayoun, R., J. Tergny and D. Keuneman. Mathematical programming with multi-objective functions: A solution by P. O. P. (progressive orientation procedure). Metra 9(2):279-299.

Boyd, D. 1970. A methodology for analyzing decision problems involving complex preference assessments. Ph.D. Dissertation. Stanford University.

Candler, Wilfred and Michael Boehlje. 1971. Use of linear programming in capital budgeting with multiple goals. Am. J. Agr. Econ. 53(2).

Charnes, A., R. W. Clower and K. O. Kortanek. 1967. Effective control through coherent decentralization with preemptive goals. Econometrica 35(2).

Charnes, A., W. W. Cooper, *et al.* 1968. Note on an application of a goal programming model for media planning. Management Science, Vol. 14, No. 8. pp. 431-436.

Charnes, A. and W. W. Cooper. 1961. Management models and industrial applications of linear programming: I and II. John Wiley and Sons, Inc., New York.

Charnes, A. and W. W. Cooper. 1968. A goal programming model for media planning. Manage. Sci., April. Vol. 14(8).

Charnes, A. and W. W. Cooper. 1969. An extended goal programming model for manpower planning. Manage. Sci. Res. Paper No. 188. Carnegie-Mellon University.

Charnes, A. and W. W. Cooper. 1971. Multi-level models for career management and resource planning. Manage. Sci. Report 256. Grad. School Indust. Admin., Carnegie-Mellon Univ., Pittsburgh. 37 p.

Charnes, A., W. W. Cooper, J. K. Devoe, D. B. Learner, and W. Reinecke. 1968. A goal programming model for media planning. Manage. Sci. Vol. 14, No. 8. 13 p.

Charnes, A., W. W. Cooper and R. O. Ferguson. 1955. Optimal estimation of executive compensation by linear programming. Manage. Sci. 1(2).

Charnes, A., W. W. Cooper and Y. Ijiri. 1963. Breakeven budgeting and programming to goals. J. Accounting Res., Vol. 1.

Charnes, A., W. W. Cooper and R. J. Niehaus. 1968. A goal programming model for manpower planning. Manage. Sci., Rep. No. 15, Carnegie-Mellon Univ. pp. 79-93.

Charnes, A., W. W. Cooper and R. J. Niehaus. 1968. Note on an application of a goal programming model for media planning. Manage. Sci. 14:B431-436.

Charnes, A., W. W. Cooper, and R. J. Niehaus. 1971. Mathematical models for manpower and personnel planning. Manage. Sci. Res. Rep. 234. Grad. School Indust. Admin., Carnegie-Mellon Univ., Pittsburgh. 32 p.

Charnes, A., W. W. Cooper, R. J. Niehaus and D. Sholtz. 1969. An extended goal programming model for manpower planning. Manage. Sci. Res. Rep. 188. Grad. School Indust. Admin., Carnegie-Mellon Univ., Pittsburgh. 11 p.

Charnes, A., W. W. Cooper, R. J. Niehaus and A. Stedry. 1969. Static and dynamic assignment models with multiple objectives and some remarks on organizational design. Manage. Sci. 4(3).

Clayton, Edward R. and Laurence J. Moore. 1972. Goal vs linear programming. J. Systems Manage., November. Vol. 18.

Contini, B. 1968. A stochastic approach to goal programming. Operations Res. 16:576-586.

Courtney, J. F., Jr., J. D. Klastorin, and T. W. Ruefli. 1972. A goal programming approach to urban-suburban location preferences. Manage. Sci. 18:258-268.

Dyer, J. S. 1971. An empirical investigation of a man-machine interactive approach to the solution of the multiple criteria problem. Discussion Paper. Western Management Science Institute, Univ. of Calif., Los Angeles.

Dyer, J. S. 1972. Interactive goal programming. Manage. Sci. 19(1):62-70.

Dyer, J. S. 1973. A procedure for selecting educational goal areas for emphasis. Operations Res. 21(3):835-844.

Elion, Samule. 1971. Goals and constraints. J. Manage. Studies 7-8:292-303.

Feinberg, A. 1972. Experimental results with an interactive multi-criterion optimization procedure. Paper presented at Tims XIX International meeting, Houston, Texas, April 5. 28 p.

Field, David B. 1973. Goal programming for forest management. Forest Sci. 19(2):125-134.

Field, David B. 1973. Forestry applications of goal programming - two examples. Working Paper for "Goal Programming For Forest Management", Forest Sci. 19(2):125-134.

Forsyth, John P. 1969. Utilization of goal programming in production and capital expenditure planning. J. Canadian O.R. Soc. 7(2).

Geoffrion, A.M., J. Dyer and A. Feinberg. 1972. An interactive approach for multi-criterion optimization, with an application to the operation of an academic department. Working Paper No. 167. Western Manage. Sci. Inst., Univ. of Calif., Los Angeles. Pages 357-368.

Gibbs, Thomas E. 1973. Goal programming. J. Systems Manage. 24(5):38-41.

Haimes, Yacov Y. and Donald Macko. 1973. Hierarchial structures in water resources systems management. IEEE Transactions on Systems, Man, and Cybernetics. Vol. SMC-3, No. 4:396-402.

Ijiri, Y. 1965. Management goals and accounting for control. Rand-McNally, Chicago.

Jaaskelainen, V. 1972. A goal programming model of aggregate production planning. Swedish J. Econ, Vol. 1(2):14-29.

Johnsen, E. 1968. Studies in multiobjective decision models. Mono. No. 1, Econ. Res. Center in Lund.

Ladd, Edward Ray. 1973. A dynamic programming approach to goal programming. Ph.D., Colorado State University.

Lane, M. N. 1970. Goal programming and satisfying models in economic analysis. Ph.D. Dissertation, Univ. of Texas, Austin.

Lee, S. M. 1970. Goal programming for management decision. Decision Sciences (Proceedings of Second Annual Meeting, AIDS). pp. 304-309.

Lee, S. M. 1971. Decision analysis through goal programming. Decision Sci. 2(2). Pages 172-180.

Lee, S. M. 1971. Decision analysis through goal programming: An ordinal solution. Decision Sci. 2(3):377-378.

Lee, S. M. 1971. Goal programming - a tutorial. A working paper. Virginia Polytechnic Institute. Pages 1-8.

Lee, S. M. 1971. An aggregative budget planning model for health care clinics. Paper presented at the 12th American Meeting of the Institute of Management Science, September. (unpublished)

Lee, S. M. 1972-73. Goal programming for decision analysis of multiple objectives. Sloan Manage. Rev., Winter. Pages 11-24.

Lee, S. M. 1972. Goal programming for decision analysis. Auerback Publishers, Princeton, New Jersey.

Lee, S. M. 1972. An aggregative resource allocation model for hospital administration. Working paper. Virginia Polytechnic Institute. Page 1-23.

Lee, S. M. 1972. A goal programming model for corporate financial decisions. (unpublished)

Lee, S. M. and M. Bird. 1970. A goal programming model for sales effort allocation. Business Perspectives 6(4):17-21.

Lee, S. M. and E. Clayton. 1969. A mathematical programming model for academic planning. Paper presented at the Southern Management Association Meeting, St. Louis.

Lee, S. M. and E. Clayton. 1970. Application of goal programming for the textile industry. Review of Indust. Manage. 6(1):107-113.

Lee, S. M. and E. R. Clayton. 1972. A goal programming model for academic resource allocation. Manage. Sci. 18(8):395-408.

Lee, S. M. and V. Jaaskelainen. 1971. Goal programming: Management's math model. Indust. Engineering, January. Vol. XIX.

Lee, S. M. and V. Jaaskelainen. 1972. Goal programming for financial planning. Liiketaloudellinen Aikakauskirja (The Finish Journal of Business Economics) 3(3):291-303.

Lee, S. M. and A. J. Lerro. 1974. Optimizing portfolio selection for mutual funds. Journal of Finance. (unpublished)

Lee, S. M., A. Lerro and B. McGinnis. 1971. Optimizaticn of tax switching for commercial banks. J. Money, Credit, and Banking 3(2):293-303.

Lee, S. M. and L. J. Moore. 1974. A model for university admissions planning. Decision Sciences. (unpublished)

Lee, S. M. and William R. Seuebeck. 1971. An aggregative model for municipal economic planning. Policy Sciences 2:99-115.

Leven, Charles L. 1964. Establishing goals for regional economic development. J. Amer. Instit. of Planners 30(2):11-109.

Marvin, Philip. 1971. Synergistic goal setting. Developing Decisions for Action, Dow Jones-Irwin, Inc., Homewood, Ill. 15 p.

McMillan, C. and F. Martinson. 1972. Goal programming and dominance among competing goals. Manage. Sci. Rep. 72-4. Graduate School of Business Admin., Univ. of Colo., Boulder.

Osteryoung, Jerome S. 1970. Multiple goals in the capital budgeting decision. Working Copy. Finance Department, Univ. South Carolina. Pages 1-13.

Pattawaik, Prasanta K. 1973. Group choice with lexicographic individual ordering. Behavioral Sci. 18. March.

Pitkanen, Lero. 1970. Goal programming and operational objectives in public administration. Swedish J. Econ. 72(3):207-214.

Price, W. L. and W. G. Piskor. 1972. The application of goal programming to manpower planning. Infor 10(3).

Roy, B. 1971. Problems and methods with multiple objective functions. Mathematical Programming 1(2):239-266.

Ruefli, T. W. 1971. A generalized goal decomposition model. Manage. Sci. 17(8).

Ruefli, T. 1971. Behavioral externalities in decentralized organizations. Manage. Sci. 17(10):649-657.

Sandell, D. A. 1972. A goal programming model for semi-controlled student flow in carbe scale educational systems. Ph.D. Dissertation, Rensselaer Polytechnic Institute.

Sayeki, Y. and Karl H. Vesper. 1973. Allocation of importance in a hierarchial goal structure. Manage. Sci. 19(6).

Scott, Allen J. 1972. Operational analysis of nodal hierarchics in network systems. Operational Res. Quart. 22(1).

Spivey, W. Allen and Hirokuni Tamura. 1971. Goal programming in econometrics. Res. Logistics Quart. 17:183-193.

Stephenson, G. G. 1971. A hierarchy of models for planning in a division of I.C.I. Operational Res. Quart. 21(2).

Suestka, Joseph A. 1966. The development of a rational group ordering technique without the use of cardinal utilities. Masters thesis, Univ. Alabama, Huntsville.

Thrall, R. M., C. H. Coombs and R. L. Davis. 1960. Decision processes. John Wiley & Sons, New York.

Wagner, H. M. 1971. Principles of management science with applications to executive decisions. Prentice-Hall, Englewood Cliffs, N.J. 562 p.

Wardle, P. A. 1965. Forest management and operations research: A linear programming study. Manage. Sci. 11(12):260-270.

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